# The riddle

We have a 10 watt laser that is programmed with an Arduino for a minimum power (duty ratio) and frequency.

- Laser wave length 444 nm
- 1Hz
- 1 duty ratio.
- When we aim the beam at an anodized plate we can hear a sound.
- When we turn it on a max power there is no audible sound.
- When we aim the beam at a steel plate on a minimum power no sound.

The question: why does it happen?

## Simple explanation

At a known frequency (1Hz) a point on the anodized aluminum plate targeted by the laser beam gets hot (and cools down by dissipation). That heat dilates that small surface. When it cools down (half cycle, when the laser is off) that surface contracts. Dilation and contraction produce vibration of the plate that depending on its dimensions and characteristics can be audible (20Hz to 20KHz). All the plates tested vibrate, at audible frequency (anodized aluminum) or inaudible (steel plate).

# Scientific explanation

Photoacoustic (PA) measurement techniques are measurement techniques where the combined interaction of light and sound are used to characterize a material under study.

Generally, the photoacoustic technique rests on an energy conversion process where electromagnetic waves are converted to mechanical waves. For a brief and simplified description of the process, assume that a short light-pulse is let to irradiate a region of a material under study. Some of the light is absorbed by the material and an associated heating process takes place. This process causes a volume change and consequently a rapid change in pressure. Since the incident light is a short pulse the conversion process will result in a pressure wave propagating through the material. The light-induced pressure wave can then be detected at some other location using some sonic detection techniques. Electromagnetic excitation sources for photoacoustics that have been used over the years are e.g. pulsed lasers in the visible-, near infrared and infrared wavelengths.

From the perspective of measurement science, both light, the photoacoustic process and the sonic waves can be used in characterizing a material in which the process takes place. The light can be used to determine the spatial distribution of optical properties in the material. For example, the strength of the photoacoustic light-to-sound conversion process at a specific point in the material under examination will depend on the amount of optical absorption at that point. Thus, a volume with high optical absorption will act as a stronger source of sound than the surrounding material. Also, the photoacoustically generated sound that travels through the material under investigation inherently carries information on the spatial distribution of acoustical properties, as distributions of density and elasticity of the material under investigation.

To gain more knowledge about what happens in the material by the thermoelastic process and about the sound that is rendered, the process can be theoretically modelled.. A theoretical model can also constitute a basis in shaping the generated sound regarding temporal profile or frequency spectrum.

#### History

Various applications of PA measurement techniques as implementations of PA physics for characterization of materials by means of light and sound, have been investigated and suggested. Foremost are applications in diagnostics for biomedical purposes, but also as example for industrial materials.

In 1880 Alexander Graham Bell reported on sound produced by light. That discovery came to use in PA techniques after inventions and developments of ultrasonic transducers, computers and lasers. The related ultrasonography (sound imaging), which is based on sonar, had been introduced in medical applications after World War II. Today, photoacoustic imaging are emerging techniques for biomedical applications.

### **Photoacoustic physics**

Photoacoustic physics studied is based on the thermoelastic effect. In the thermoelastic process, e.g. a short pulse of light, is sent into a material, and energy absorbed from the light induces a rapid heating process and an associated pressure rise. Other processes that can give rise to sound from light are plasma formation, vaporisation, ablation, chemical reactions and electrostriction, depending on the EM wavelength  $\lambda$  and power, in interaction with a material under investigation. Vaporisation is when the physical phase of a substance, by means of energy added, is changed into vapour. Ablating the top of the surface of a sample by means of a laser can also give sound. Plasma formation is when the substance is ionized into plasma phase. When chemical reactions are induced by light as laser, and the reaction products takes up different space than the initial constituents, a pressure wave might be formed. Electrostriction is a property of dielectric materials (electrical non-conductors) to change in shape under an applied electric field. In the electric field, randomly aligned electrical domains become differently charged, attracting or repelling each other. An effect related to electrostriction is piezoelectricity. For energy input for thermoelastic effect, the light source can for example be a pulsed laser or an amplitude modulated continuous laser. For this case we have a pulsed laser. The aspect of variation in heating is of importance in the photoacoustics.



*Figure 1: Heating of a material volume leading to a volume expansion.* 

## PA instant pressure

As an introduction into, and to deduce an expression for an instant PA pressure, material volume V = V(p',T') as a function of pressure p' and temperature T' can be expressed as differential

$$dV = dp' \left(\frac{\partial V}{\partial p'}\right)_{T'} + dT' \left(\frac{\partial V}{\partial T'}\right)_{p'} \tag{1}$$

where the subscripts *T'* and *p'* means differentiation in one variable while the other in subscript is held constant. Inserting in equation 1 thermal expansion coefficient  $\beta$  and isothermal compressibility  $\kappa$ 

$$\kappa = -\frac{1}{V} \left( \frac{\partial V}{\partial p'} \right)_{T'}, \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T'} \right)_{p'}$$
(2)

gives

$$dV = -dp' \kappa V + dT' \beta V.$$
(3)

Instead defining pressure and temperature changes as

dp' = p

and

dT' = T,

a PA fractional volume expansion *dV/V* on laser excitation can be expressed

$$\frac{dV}{V} = -\kappa \, p + \beta \, T. \tag{4}$$

Figure 1 illustrates heating of a material volume leading to a thermoelastic expansion. If the duration of the heating is short, a local pressure wave is built up. This acoustic pressure wave can propagate through the heated material with surroundings as a sonic

pulse which can be detected by a sound transducer and subsequently analyzed. If the duration of the heating is much shorter than the time of heat conduction as well as of pressure propagation across a size of the heated region, the fractional volume expansion is assumed negligible and the local pressure change  $p_{instant}$  instantly after the heating laser pulse, can be formulated from equation 4 as

$$p_{instant} = \frac{\beta T}{\kappa},\tag{5}$$

where then pressure and temperature changes are proportional to each other. To describe pressure not only instantaneously, but from a heating process and in time and space, a PA wave equation follows.

### PA wave equation

Originating in conservation of energy and of momentum, the generation and propagation of heat-induced sound, can be described by a PA wave equation, expressing sound pulses as acoustic pressure  $p = p(\mathbf{x}, t)$  in position  $\mathbf{x}$  at time t. The thermal expansion equation, a generalization of Hooke's law, implies conservation of energy, with requirement of linearly elastic material, and is a 3D vector expressed version of the already introduced equation 4

$$\nabla \cdot \boldsymbol{\xi} \left( \mathbf{x}, t \right) = -\kappa p(\mathbf{x}, t) + \beta T(\mathbf{x}, t)$$
(6)

with medium displacement vector  $\xi$  (**x**,*t*). Conservation of momentum is represented by the linear inviscid force equation, from Newtons second law, as

$$\rho \frac{\partial^2}{\partial t^2} \boldsymbol{\xi} \left( \mathbf{x}, t \right) = -\nabla p(\mathbf{x}, t)$$
(7)

where  $\rho$  is the material density.

The two equations 6 and 7 make up the basis for the PA wave equation in a linearly elastic inviscid material that is fluid or solid with only longitudinal waves considered. Inserting equation 6 into the divergence of equation 7, setting speed of sound c as

 $c^2 = 1/(\rho\kappa)$  gives a general PA wave equation in an inviscid linearly elastic medium

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) p(\mathbf{x}, t) = -\frac{\beta}{\kappa c^2} \frac{\partial^2 T(\mathbf{x}, t)}{\partial t^2}$$
(8)

where the right hand side constitutes a source term for the left hand side acoustic wave. The speed of sound in the material gives the pressure wave propagation speed in the material. Also included in equation 8 is the thermal expansion coefficient, temperature change, and isothermal compressibility

$$\kappa = \frac{C_p}{\rho \, c^2 \, C_{V_p}} \tag{9}$$

where  $C_p$  is specific heat capacity at constant pressure, and  $C_V$  is specific heat capacity at constant volume.

To start from the beginning in the light-to-sound conversion in a material, light interaction with materials can be represented by extinction coefficient  $\mu_t$ 

$$\mu_t = \mu_a + \mu_s \tag{10}$$

where  $\mu_a$  is absorption coefficient and  $\mu_s$  scattering coefficient. These coefficients are measured in m<sup>-1</sup> as a probability of photon interaction per unit path length of the light trajectory. Figure 2 illustrates an overview of PA light-to-sound conversion in a sample. **Light with wavelength**  $\lambda$ , **pulse length**  $\tau_{pulse}$  **and energy**  $E_0$ , **is irradiated into and interacts with the materials in a sample that is having optical properties as absorption and scattering coefficients**. The sample thermal material properties as thermal expansion coefficient and heat capacities affect the thermoelastic conversion, and mechanical properties, as sound speed and density, further influence the generated sound. Because of the sound generation in and interaction with the sample, the sound carries information that can be extracted about the sample regarding material properties and distributions of material properties within the sample.



Figure 2: Overview of photoacoustic light-to-sound conversion in a sample. The red oval symbolises a light-absorbing feature.

Something to consider before further PA modelling is heat conduction during the heating of duration  $\tau$ pulse. The thermal relaxation time  $\tau$ thermal for thermal diffusion across a characteristic lenght of dimension dc was written by Wang and Wu as

$$\tau_{thermal} = \frac{d_c^2}{\alpha_{thermal}} \tag{11}$$

where  $\alpha_{thermal}$  is thermal diffusivity from Gusev and Karabutov as

$$\alpha_{thermal} = \frac{\lambda_{thermal}}{\rho C_p} \tag{12}$$

with thermal conductivity  $\lambda_{thermal.}$ 

With negligible heat conduction during the heating,  $\tau_{thermal} \gg \tau_{pulse}$ , called thermal confinement, and heating function  $H(\mathbf{x}, t)$  as energy absorbed per time per unit volume, the right hand side of equation 8 can be rewritten by means of

$$\rho C_V \frac{\partial T\left(\mathbf{x}, t\right)}{\partial t} = H\left(\mathbf{x}, t\right)$$
(13)

Equation 13 comes from the heat conduction equation

$$\rho C_V \frac{\partial T(\mathbf{x}, t)}{\partial t} = \lambda_{thermal} \nabla^2 T + H(\mathbf{x}, t)$$
(14)

describing the temperature distribution from generated heat, but with the term in equation 14 including thermal conductivity  $\lambda_{thermal}$  neglected for thermal confinement.

A PA wave equation in thermal confinement modelling generation and propagation of acoustic pressure  $p(\mathbf{x},t)$  is then

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) p(\mathbf{x}, t) = -\frac{\beta}{C_p} \frac{\partial H(\mathbf{x}, t)}{\partial t}$$
(15)

In the right hand side of PA wave equation 15, the PA source term contains the time derivative of the heating. Theoretically and in fluid, an ideal thermoelastic source produces a pressure profile proportional to the time derivative of the heating from the laser pulse.

It is thereby the change in the heating that gives rise to the induced sound. The position **x** can be a vector of two or three dimensions, but in this explanation a 1D scalar **x** is used. Not included in equation 15 is e.g. sound attenuation in the medium.

A factor to mention in the context, for later reference, is Gruneisen parameter

$$\Gamma_i = \frac{c_i^2 \beta_i}{C_{p\,i}} = \frac{\beta_i}{\kappa_i \,\rho_i \,C_{V\,i}},\tag{16}$$

which is dimensionless, temperature-dependent, and proportional to the fraction of thermal energy converted into acoustic pressure, where index *i* denotes different material layers *i*. PA initial acoustic pressure distribution in one dimension *x* can be written from Gusev and Karabutov by means of Gruneisen parameter as

$$p_i(x,0) = \mu_{a\,i} \, E_{0\,i} \, \Gamma_i \, \mathrm{e}^{-\mu_{a\,i}x}. \tag{17}$$

Light scattering  $\mu_s$  was neglected in comparison to light absorption  $\mu_a$  in the calculations of analytical photoacoustic expressions.

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